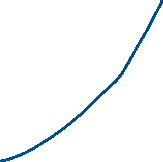
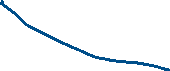
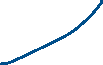
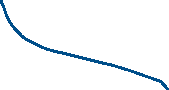
1.

a) i) f is defined on a domain without boundary, so the FONC becomes f’(x)=0. Since this is not the case for any x, there are no extrema of f.

ii) we now have a bounded domain. We cannot conclude anything about global extrema, because we don’t know f(-a) and f(a), which could be the global extrema. Furthermore, we don’t know if f’(0) exists, so we cannot conclude that it is zero there.

Consider the below diagrams:



Notice that this gives no indication on whether *f* is a maximum or a minimum, as the diagrams show.

b) this is pretty much the same as in tut 3, Q2. Only difference is instead of , which makes us obtain instead of .

2.

a) I)

The eigenvalues are the λ for which (this is the characteristic polynomial). This leads to lambda = (2+-beta) >= 0 if –2<=beta<=2. And otherwise the two values of lambda will be symmetric around zeros, one being smaller and one larger than zero, so the Hessian is indefinite.

ii) I solved grad f = 0 and obtained:

X = -(beta-1)^2 and y=beta-2

We want . This means that

This means that and . We now need to write *x* and *y* in terms of β.

Notice that . Then, given that , we have

Similarly for *y*,

This would be the set of points that satisfy the FONC, as long as .

(iii) Work with the SOSC for this one. TODO for the solution. Hint: the Hessian.

For the –2<=beta<=2, we know from I) that f is convex so it should be a global minimum, because the function is convex. For all other beta, we know that f is indefinite, so we should have saddle points I think

iv) ??

b) necessary condition: grad g2 = 0

Sufficient condition: the previous one and Hessian g2 > 0

Taking derivatives, we get

grad g2 = a\*grad h

Hessian g2 = a\* Hessian h

Therefore we need h to be convex in this case

3.

a)

Necessary:

lambda\* >=0

H(x)<=0

Grad f + lambda\*grad h = 0

Lambda\*h(x) = 0

Sufficient: the necessary holds and the Hessian is convex

We know that f and the feasible region are convex, so we have a convex optimisation problem for which we can follow that if it fulfills the necessary conditions, the point x\* will be a global unique minimum.

b) Looking at the constraints we see it’s the same constraints as in the example in the lecture pretty much. If we do the maths, we can find that the constraints are only both fulfilled for (0, 0). Then the derivatives of the two constraints are (-2, 0) and (-4, 0), which are linearly dependent so there are no regular points that satisfy the constraints.

c) we get x\_1 + lamba = 0, and the same for x2 and x3 and the constraint. From this we see that x1=x2=x3=-lamda. Deriving the hessian, we get the identity (I think?), so we know it is an extremum. Also, we can see that the hessian of our function that we minimise is the identity, so it is a global minimum

d) we can see quite easily that the function that we minimise decreases without bound. We can also construct any desired value by using the solution x1=n, x2=1-n, x3=0, which for n->infinity will make f(x1,x2,x3)->-infinity

4. A significant portion of this year’s coursework was recycled from this question (which is an adaption of the original question from which the coursework question was taken, namely the 2014-15 exam).

OMG!